

# Conditions of Low Dimensionality for Strongly Interacting Atoms Under a Transverse Trap

J. P. Kestner, L.-M. Duan

*FOCUS center and MCTP, Department of Physics, University of Michigan, Ann Arbor, MI 48109*

For a dilute atomic gas in a strong transverse trapping potential, one normally expects that, in the ground state, the gas will populate only the lowest transverse level. We show, however, that for the strongly interacting gas under a Feshbach resonance, the ground state includes a large fraction of atoms in excited levels of the trap, even if the gas is very dilute and the trap is very strong. This is because the effective atom-molecule coupling is typically enhanced to many times the trap mode spacing by an induced confinement along the untrapped dimension(s). Thus one cannot "freeze out" the transverse degrees of freedom except under certain conditions.

## I. INTRODUCTION

Recently, the system of interacting atomic gas at low dimensions has attracted considerable interest as it supports a wealth of physics [1, 2, 3, 4, 5, 6, 7]. For ultracold atoms, low dimensions are typically achieved by applying a transverse optical lattice potential [8]. With a one (or two) dimensional deep optical lattice potential, the atoms trapped at different potential wells basically do not interact with each other. For weakly interacting gas, one can then freeze out the transverse degrees of freedom by assuming the system to be in the lowest level of the trap potential. The atomic gas at each individual potential well thus behaves as an effective two (or one) dimensional system.

The condition for achieving effective low-dimensionality in an optical lattice becomes more involved if one has strongly interacting atomic gas. One of the most exciting directions in current atomic physics is to study this strongly interacting atomic gas, where the effective interaction as measured by the atomic scattering length can be tuned in the full range via a Feshbach resonance through control of an external magnetic field [9]. The Feshbach resonance basically comes from the coupling of atoms in the open collision channel to the molecules in the closed channel. If this atom-molecule coupling rate becomes larger than the level spacing of the transverse trapping potential, one cannot assume a low-dimensional system by freezing the transverse mode in the lowest level [10, 11]. On the other hand, intuitively, one may expect that the effective atom-molecule coupling rate always decreases when we make the gas more dilute, and the transverse level spacing increases when we raise the trapping laser intensity. So, for a sufficiently dilute gas under a strong transverse trap, one can still get the transverse level spacing larger than the atom-molecule coupling rate. With this expectation, several recent works have studied properties of low-dimensional strongly interacting gas by (implicitly) assuming the transverse mode in the trap ground state [5, 6].

In this work, we want to show that the condition of low dimensionality for strongly interacting atoms under a transverse trap is more subtle than the above simple

picture. We will show that even if the gas becomes very dilute (in the limiting case, one can just have two atoms with the gas density tending to zero), for the ground state of the system, we will still have a significant (actually, dominant) fraction of atomic population in the excited transverse levels under a typical wide Feshbach resonance. Furthermore, it is very ineffective to reduce the transverse excited fraction by increasing the trapping laser intensity. For realistic atoms such as  $^{40}\text{K}$  or  $^6\text{Li}$ , even if the trapping potential is increased to some completely impractical level, the transverse excited fraction is not yet negligible. The basic reason for this unusual phenomena is that in the low-dimensional trap, there is an increased tendency for atoms to pair up spatially also along the untrapped dimension(s). This effect lends itself to an intuitive understanding of the coupling enhancement in the dilute limit. Furthermore, with stronger transverse traps, the atoms in the induced pairs will become more spatially confined also along the untrapped dimension(s). So, although the transverse level spacing increases, the effective atom-molecule coupling is also significantly enhanced. As a net effect, the atomic population in the transverse excited levels becomes pretty insensitive to the magnitude of the trapping potential. Although the result here does not preclude the possibility of an effective low-dimensional Hamiltonian for strongly interacting atoms under the transverse trap, it indeed shows that in general, one can not neglect the atomic population in the transverse excited levels, and derivation of an effective low-dimensional Hamiltonian would be more subtle and tricky than one naively expects [10, 12].

In the next section, we give the general formalism for strongly interacting atoms under a transverse trap in the dilute gas limit. In that limit, the basic picture is captured by two-body physics. For the ground state of the system, the atoms form pairs, and interaction between the pairs become negligible. There have been a few works on description of two-body physics of strongly interacting atoms in a trap using the single-channel model [7]. Recently, there have also been descriptions of the problem with a more realistic two-channel model for the Feshbach resonance (in one-dimensional or three-dimensional traps [11, 13]). The formalism there, however, neglects the atomic background scattering. We extend this for-

malism to include the background scattering, which becomes necessary when the system is outside of the near-resonance region. In Sec. III, we present our main calculation results for the condition of low-dimensionality under a transverse one-dimensional or two-dimensional traps. We will also give detailed studies of the atomic population distributions in the transverse levels and in the free dimension(s) for the  $^{40}\text{K}$  and  $^6\text{Li}$  atoms under trapping potentials of various intensities.

## II. GENERAL FORMALISM

To achieve a  $d$ -dimensional ( $d < 3$ ) atomic gas, we assume a  $(3-d)$ -dimensional optical lattice applied along the transverse direction. The lattice potential barrier is high so that the atomic interaction between different potential wells becomes negligible. With a strong lattice potential, the atoms at the bottom of the potential wells basically see a transverse harmonic trap. The atomic gas in each well can then be modeled as in a  $(3-d)$ -dimensional harmonic trap of a characteristic trapping frequency  $\omega$ . The atoms are of mass  $m$  and possess internal states  $\sigma = \{\uparrow, \downarrow\}$ . We treat the problem of strongly interacting atoms across a Feshbach resonance using the standard two-channel model [14]. The short range interaction between closed-channel molecules and open-channel atoms is modeled by a delta function. The Hamiltonian is expressed as  $H = H_0 + H_I$ , with

$$H_0 = \sum_{\sigma=\uparrow,\downarrow} \int d^3\mathbf{r} \Psi_{\sigma}^{\dagger} \left( -\frac{\hbar^2 \nabla^2}{2m} + \frac{1}{2} m \omega^2 \sum_{i=1}^{3-d} x_i^2 \right) \Psi_{\sigma} \\ + \int d^3\mathbf{r} \Phi^{\dagger} \left( -\frac{\hbar^2 \nabla^2}{4m} + m \omega^2 \sum_{i=1}^{3-d} x_i^2 + \bar{\nu}_b \right) \Phi$$

and

$$H_I = \bar{g}_b \int d^3\mathbf{r} \left( \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Phi + h.c. \right) + \bar{U}_b \int d^3\mathbf{r} \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Psi_{\downarrow} \Psi_{\uparrow}, \quad (1)$$

where  $\Psi(\mathbf{r})$  is the atomic field operator,  $\Phi(\mathbf{r})$  is the molecular field operator,  $\bar{\nu}_b$  is the bare detuning,  $\bar{g}_b$  is the bare atom-molecule coupling rate, and  $\bar{U}_b$  is the bare background atomic scattering rate. The bare parameters are related to the physical parameters via the standard renormalization relations [15]:

$$\bar{U}_c^{-1} = - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\epsilon_{\mathbf{k}}}, \quad \Gamma^{-1} = 1 + \frac{\bar{U}_p}{U_c} \\ \bar{U}_b = \Gamma \bar{U}_p, \quad \bar{g}_b = \Gamma \bar{g}_p, \quad \bar{\nu}_p = \bar{\nu}_b + \Gamma \frac{\bar{g}_p^2}{U_c} \quad (2)$$

where the subscript  $p$  denotes physical parameters,  $\epsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / 2m$ , and the integral is taken in three dimensions with an explicit energy cutoff  $E_c$  imposed on two dimensions [16]. Then  $\bar{U}_c^{-1} = m^{3/2} \sqrt{E_c} / 2^{3/2} \pi \hbar^3$ . The physical

parameters  $\bar{g}_p, \bar{U}_p, \bar{\nu}_p$  are determined from the scattering data as  $\bar{U}_p = 4\pi \hbar^2 a_{bg} / m$ ,  $\bar{g}_p = \sqrt{4\pi \hbar^2 \mu_{co} W |a_{bg}|} / m$ , and  $\bar{\nu}_p = \mu_{co} (B - B_0)$  ( $\mu_{co}$  is the difference in magnetic moments between the two channels) [10], where we have assumed that the s-wave scattering length near resonance has the form  $a_s = a_{bg} \left( 1 - \frac{W}{B-B_0} \right)$ , with  $a_{bg}$  as the background scattering length,  $W$  as the resonance width, and  $B_0$  as the resonance point.

We assume the gas to be sufficiently dilute so that we need consider only two-body physics within each potential well. At a very low temperature, two atoms interact and form bound atom-pairs. The interaction between the atom pairs is negligible in the limit of a very dilute gas. The essential physics is then captured by considering the state of two atoms under the above interaction Hamiltonian. For the two-body physics, the center-of-mass degree of freedom is not influenced by the interaction and can be separated under a harmonic potential. So we can assume the center-of-mass mode is in the ground state of the transverse trap and has zero momentum in the free dimension(s). In this center-of-mass frame, expanding the field operators  $\Psi(\mathbf{r})$  and  $\Phi(\mathbf{r})$  in Eq. (1) in terms of harmonic modes in the trapped dimensions and plane waves in the untrapped dimensions yields

$$H_0 = \hbar \omega \sum_{\mathbf{m}\mathbf{k}\sigma} \epsilon_{\mathbf{m}\mathbf{k}} a_{\mathbf{m}\mathbf{k}\sigma}^{\dagger} a_{\mathbf{m}\mathbf{k}\sigma} + \bar{\nu}_b b^{\dagger} b, \quad (3)$$

where  $\mathbf{m}$  indexes trap eigenmodes  $\{m_i\}$ ,  $i = 1, \dots, 3-d$ , and  $\mathbf{k}$  denotes the wave vector in the untrapped dimensions  $\{k_j\}$ ,  $j = 1, \dots, d$ . The operators  $a_{\mathbf{m}\mathbf{k}\sigma}$  and  $b$  represent the corresponding atomic and molecular modes, respectively. As there is only one molecular mode in the center-of-mass frame, we drop the index of  $b$ . We have excluded the center-of-mass energy in the Hamiltonian (4). The atom relative energy  $\epsilon_{\mathbf{m}\mathbf{k}}$  is given (in the unit of  $\hbar \omega$ ) by

$$\epsilon_{\mathbf{m}\mathbf{k}} = \frac{3-d}{4} + \sum_{i=1}^{3-d} m_i + \frac{a_t^2}{2} \sum_{j=1}^d k_j^2, \quad (4)$$

where  $a_t = \sqrt{\hbar/m\omega}$  characterizes the trap length scale. Likewise, the interaction Hamiltonian  $H_I$  in terms of these modes has the form

$$H_I = \frac{\bar{g}_b}{a_t^{(3-d)/2} L^{d/2}} \sum_{\mathbf{m}\mathbf{n}\mathbf{k}} \gamma_{\mathbf{m}\mathbf{n}} \left( a_{\mathbf{m}\mathbf{k}\uparrow}^{\dagger} a_{\mathbf{n}-\mathbf{k}\downarrow}^{\dagger} b + h.c. \right) \\ + \frac{\bar{U}_b}{a_t^{(3-d)} L^d} \sum_{\substack{\mathbf{m}\mathbf{n}\mathbf{k} \\ \mathbf{m}'\mathbf{n}'\mathbf{k}'}} \gamma_{\mathbf{m}\mathbf{n}} \gamma_{\mathbf{m}'\mathbf{n}'} a_{\mathbf{m}\mathbf{k}\uparrow}^{\dagger} a_{\mathbf{n}-\mathbf{k}\downarrow}^{\dagger} a_{\mathbf{n}'-\mathbf{k}'\downarrow} a_{\mathbf{m}'\mathbf{k}'\uparrow}, \quad (5)$$

where

$$\gamma_{\mathbf{m}\mathbf{n}} = \prod_{j=1}^{3-d} \begin{cases} \frac{(-1)^{(m_j-n_j)/2}}{(2\pi^3)^{1/4} \sqrt{m_j! n_j!}} \Gamma\left(\frac{m_j+n_j+1}{2}\right) & m_j + n_j \text{ even} \\ 0 & m_j + n_j \text{ odd} \end{cases}. \quad (6)$$

For convenience, we define dimensionless bare parameters

$$g_b = \frac{\bar{g}_b a_t^{-3/2}}{\hbar\omega}, \quad U_b = \frac{\bar{U}_b a_t^{-3}}{\hbar\omega}, \quad \nu_b = \bar{\nu}_b / \hbar\omega \quad (7)$$

and likewise for dimensionless physical parameters  $g_p$ ,  $U_p$ , and  $\nu_p$ .

A general two-body state for the atoms and the molecule can be expressed as [11]

$$|\Psi\rangle = \left( \beta b^\dagger + \sum_{\mathbf{m}\mathbf{n}\mathbf{k}} \eta_{\mathbf{m}\mathbf{n}\mathbf{k}} a_{\mathbf{m}\mathbf{k}\uparrow}^\dagger a_{\mathbf{n}-\mathbf{k}\downarrow}^\dagger \right) |vac\rangle. \quad (8)$$

The coefficients in this ansatz state (normalized to unity) are obtained by solving the Schrödinger equation  $H|\Psi\rangle = E\hbar\omega|\Psi\rangle$ , which yields

$$\frac{1}{U_b^{eff}(E)} = S(E), \quad (9)$$

$$\beta^{-2} = 1 - Z_b^2(E) \frac{\partial S(E)}{\partial E}, \quad (10)$$

$$\eta_{\mathbf{m}\mathbf{n}\mathbf{k}} = \beta \gamma_{\mathbf{m}\mathbf{n}} (a_t/L)^{d/2} \frac{Z_b(E)}{E - \epsilon_{\mathbf{m}\mathbf{k}} - \epsilon_{\mathbf{n}\mathbf{k}}}. \quad (11)$$

where

$$U_b^{eff}(E) \equiv U_b - \frac{g_b^2}{\nu_b - E}, \quad Z_b(E) \equiv g_b - \frac{U_b}{g_b} (\nu_b - E), \quad (12)$$

and

$$S(E) \equiv \left( \frac{a_t}{L} \right)^d \sum_{\mathbf{m}\mathbf{n}\mathbf{k}} \frac{\gamma_{\mathbf{m}\mathbf{n}}^2}{E - \epsilon_{\mathbf{m}\mathbf{k}} - \epsilon_{\mathbf{n}\mathbf{k}}}. \quad (13)$$

Thus, Eq. (9) determines the eigenenergy  $E$ , and Eqs. (10) and (11) give us the eigenstate as a function of the eigenenergy. These equations are expressed in terms of the bare parameters, and we need to use the renormalization relation (2) to transfer them into the ones with the physical parameters. One can easily check that under the relation (2),  $Z_p(E) \equiv g_p - \frac{U_p}{g_p} (\nu_p - E) = Z_b(E)$  for any  $E$ , and  $[U_p^{eff}(E)]^{-1} \equiv \left( U_p - \frac{g_p^2}{\nu_p - E} \right)^{-1} = [U_b^{eff}(E)]^{-1} - \bar{U}_c^{-1} a_t^3 \hbar\omega$ . The divergence in  $\bar{U}_c^{-1}$  then exactly cancels with the divergence in  $S(E)$ . As a net result, the above equations (9)–(12) retain the same form upon renormalization — all the bare parameters are replaced by their physical counterparts, and  $S(E)$  and  $\partial S(E)/\partial E$  are replaced by

$$S_p(E) = \frac{-1}{2^{5/2}\pi} \begin{cases} \zeta(1/2, 1/2 - E/2) & d = 1 \\ \int_0^\infty ds \left( \frac{\Gamma(s + \frac{1}{4} - \frac{E}{2})}{\Gamma(s + \frac{3}{4} - \frac{E}{2})} - \frac{1}{\sqrt{s}} \right) & d = 2 \end{cases} \quad (14)$$

$$\frac{\partial S_p(E)}{\partial E} = \frac{-1}{2^{7/2}\pi} \begin{cases} \frac{1}{2} \zeta(3/2, 1/2 - E/2) & d = 1 \\ \frac{\Gamma(1/4 - E/2)}{\Gamma(3/4 - E/2)} & d = 2 \end{cases} \quad (15)$$

where  $\zeta(s, x) = \lim_{N \rightarrow \infty} \sum_{n=0}^N (n+x)^{-s} - \frac{(N+x)^{-s+1}}{-s+1}$  is the Hurwitz zeta function and  $\Gamma(x)$  is the gamma function. The above set of equations serve as the basic formalism for determining the state of two atoms in a transverse trap across a Feshbach resonance. If we take the one-dimensional case ( $d = 1$ ) and neglect the background scattering (let  $U_p = 0$ ), the above equation for  $E$  is reduced to the energy equation in Ref. [13], where it is derived with a different renormalization method.

### III. TRANSVERSE POPULATION DISTRIBUTIONS FOR $^{40}\text{K}$ AND $^6\text{Li}$ AND CONDITIONS FOR LOW-DIMENSIONALITY

In this section, we answer the following basic question: For a dilute gas under a strong transverse trap, can we assume the atoms only populate the lowest transverse level at low temperature so that the system becomes effectively low-dimensional? In the extreme limit of a dilute gas, we can consider only two atoms in each potential well (the density in the free dimension(s) tends to zero in this case). We will show that even in this extreme limit the excited fraction is typically still significant (dominant) for realistic atoms. If the gas density becomes higher, the excited fraction surely cannot decrease.

To answer this question, we use the above formalism for the two-atom state, and calculate the population distribution in the transverse levels for  $^{40}\text{K}$  and  $^6\text{Li}$  atoms, which are the relevant species for the current experiments. We take the scattering parameters  $W \simeq 8G$ ,  $a_{bg} \simeq 174 a_B$ ,  $\mu_{co} \simeq 1.68\mu_B$  for  $^{40}\text{K}$  [9] and  $W \simeq 300G$ ,  $a_{bg} \simeq -1405 a_B$ ,  $\mu_{co} \simeq 2\mu_B$  for  $^6\text{Li}$  [17]. With a typical trap frequency  $\omega \simeq 2\pi \times 62 \text{ kHz}$  [3], the physical parameters for the atom-molecule interaction are then given by  $g_p = 23$  (272),  $U_p = 1.7$  (−5.5) for  $^{40}\text{K}$  ( $^6\text{Li}$ ). To calculate the transverse population distribution in the system ground state, we first determine the binding energy between the atoms using Eq. (9). The binding energy  $E_B = E - (3 - d)/2$  (in the unit of  $\hbar\omega$ ), where the latter term  $(3 - d)/2$  is the energy for two free atoms (excluding the contribution from the center-of-mass mode). As the binding energy is of interest by itself, we show  $E_B$  as a function of the magnetic field detuning  $B - B_0$  in Fig. 1(a) and 1(b) with  $d = 1, 2$  for  $^{40}\text{K}$  and  $^6\text{Li}$ , respectively. For  $^{40}\text{K}$ , the binding energy saturates on the deep BEC side. This effect comes from the positive background scattering length of  $^{40}\text{K}$ , and is related to the weakly bound state in the open collision channel. The detailed discussion of the background scattering effects can be found in Ref. [18] (see also [19]). So, outside of the near resonance region, the result here is quite different from the one in Ref. [13], where the binding energy is calculated for the  $d = 1$  case without the background scattering contribution. From Fig. 1, one can also see that with a transverse trap, the binding energy  $|E_B|$  is always positive, and only tends to zero when one goes to the deep BCS limit [3, 7, 13] (or when  $B - B_0 \rightarrow W$  for

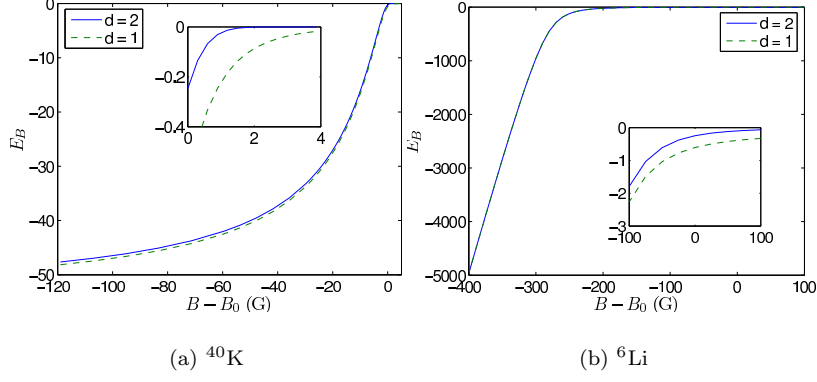


FIG. 1: Binding energy vs detuning at  $\omega \simeq 2\pi \times 62 \text{ kHz}$ .  $B_0$  is the resonant point in the absence of an optical lattice. The inserts show a close-up of the binding energy in the near-resonance region.

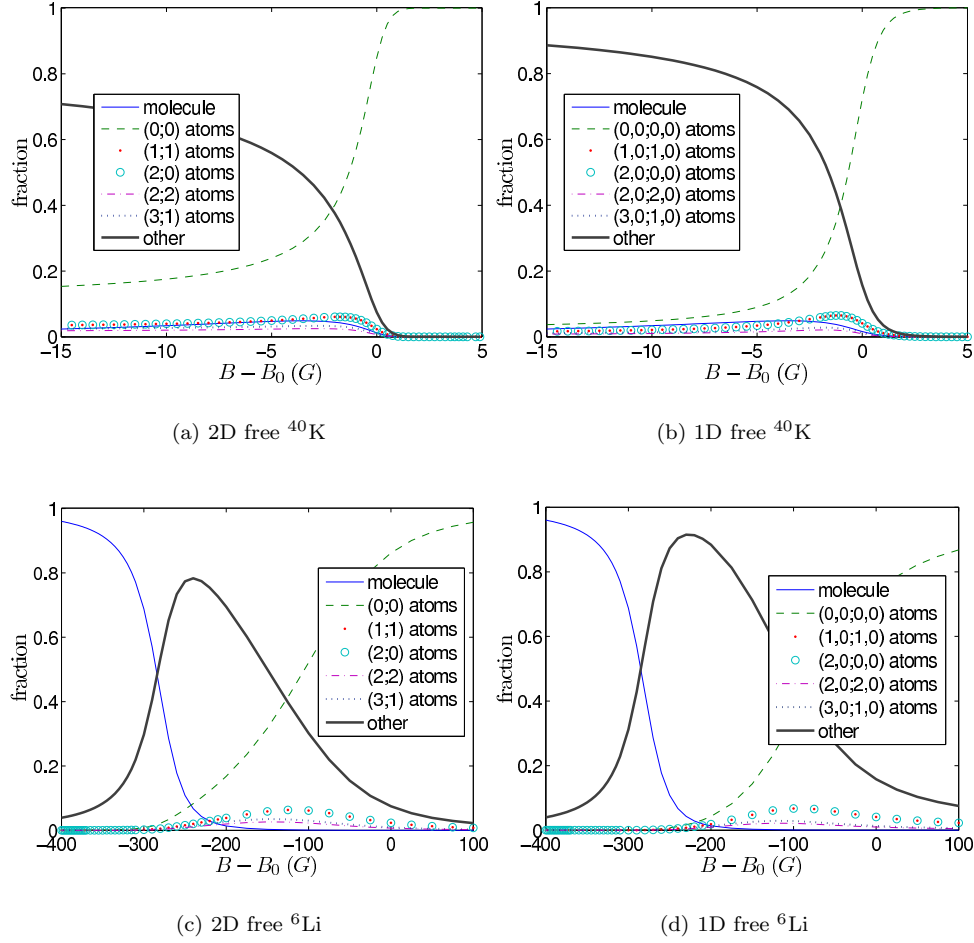
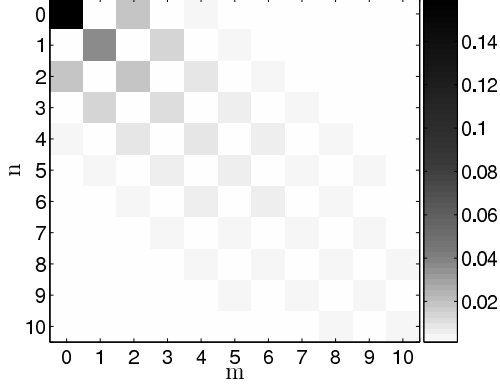
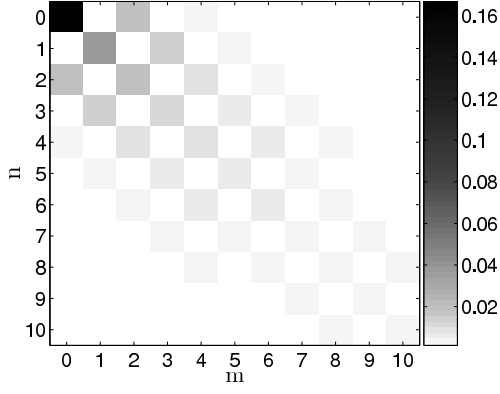


FIG. 2: Ground state composition vs. detuning. Only the six most significant components are shown explicitly. The label  $(\mathbf{m}; \mathbf{n})$  denotes the sum of  $P_{\mathbf{mn}} = \sum_{\mathbf{k}} \eta_{\mathbf{mnk}}^2$  and all components identical by symmetry.

$^{40}\text{K}$ , where the scattering length goes to zero). This is distinct from the case without traps, where  $E_B = 0$  for  $B \geq B_0$ .

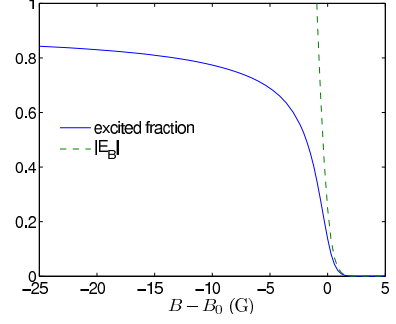
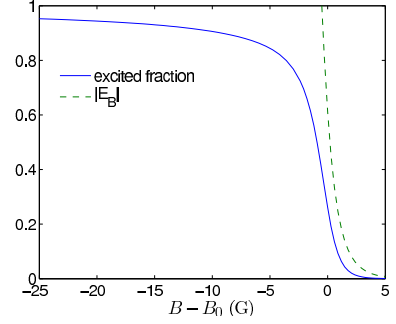
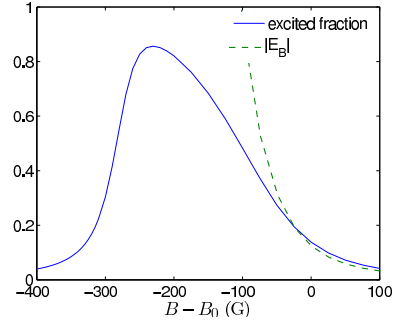
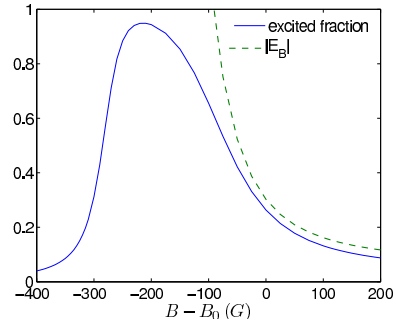
In Fig. 2, we show the population distribution in trans-

verse levels for  $^{40}\text{K}$  and  $^{6}\text{Li}$  in two or one dimensional traps as a function of the magnetic field detuning. The population fraction in the transverse modes  $(\mathbf{m}; \mathbf{n})$  is defined as  $P_{\mathbf{mn}} = \sum_{\mathbf{k}} |\eta_{\mathbf{mnk}}|^2$ . In Fig. 3, we show a more

(a)  $^{40}\text{K}$ ,  $B - B_0 = -18 \text{ G}$ (b)  $^6\text{Li}$ ,  $B - B_0 = -200 \text{ G}$ FIG. 3: The matrix  $\sum_{\mathbf{k}} \eta_{mn\mathbf{k}}^2$  for 2D free atoms on the BEC side.

complete picture of the population distribution at a fixed detuning on the BEC side for atoms in a one dimensional trap. From the figures, one can see that in general many transverse modes are populated. For a fixed mode, the population still goes down as the energy of the mode goes up, but there are so many excited transverse modes that the total population fraction in the excited levels actually dominates in typical configurations.

In Fig. 4, we draw the overall fraction populating the transverse excited levels, which is defined as  $P_{ex} \equiv 1 - \beta^2 - \sum_{\mathbf{k}} \eta_{00\mathbf{k}}^2$ . This fraction needs to satisfy  $P_{ex} \ll 1$  for the assumption that the atoms only populate the lowest transverse level. From the figure, one can see that this condition is in general not satisfied for  $^{40}\text{K}$  and  $^6\text{Li}$ , except in the deep BCS limit with the binding energy  $|E_B| \ll 1$ . For the case of  $^6\text{Li}$ , if one goes to the very deep BEC limit with the closed channel population (the bare molecule fraction)  $\beta \rightarrow 1$ , the condition  $P_{ex} \ll 1$  is also automatically satisfied. For  $^{40}\text{K}$ , because of its positive background scattering length, the bare molecule fraction  $\beta$  remains small even if one goes to the deep BEC

(a) 2D free  $^{40}\text{K}$ (b) 1D free  $^{40}\text{K}$ (c) 2D free  $^6\text{Li}$ (d) 1D free  $^6\text{Li}$ FIG. 4: Excited fraction and binding energy vs. detuning for  $\omega \simeq 2\pi \times 62 \text{ kHz}$ .

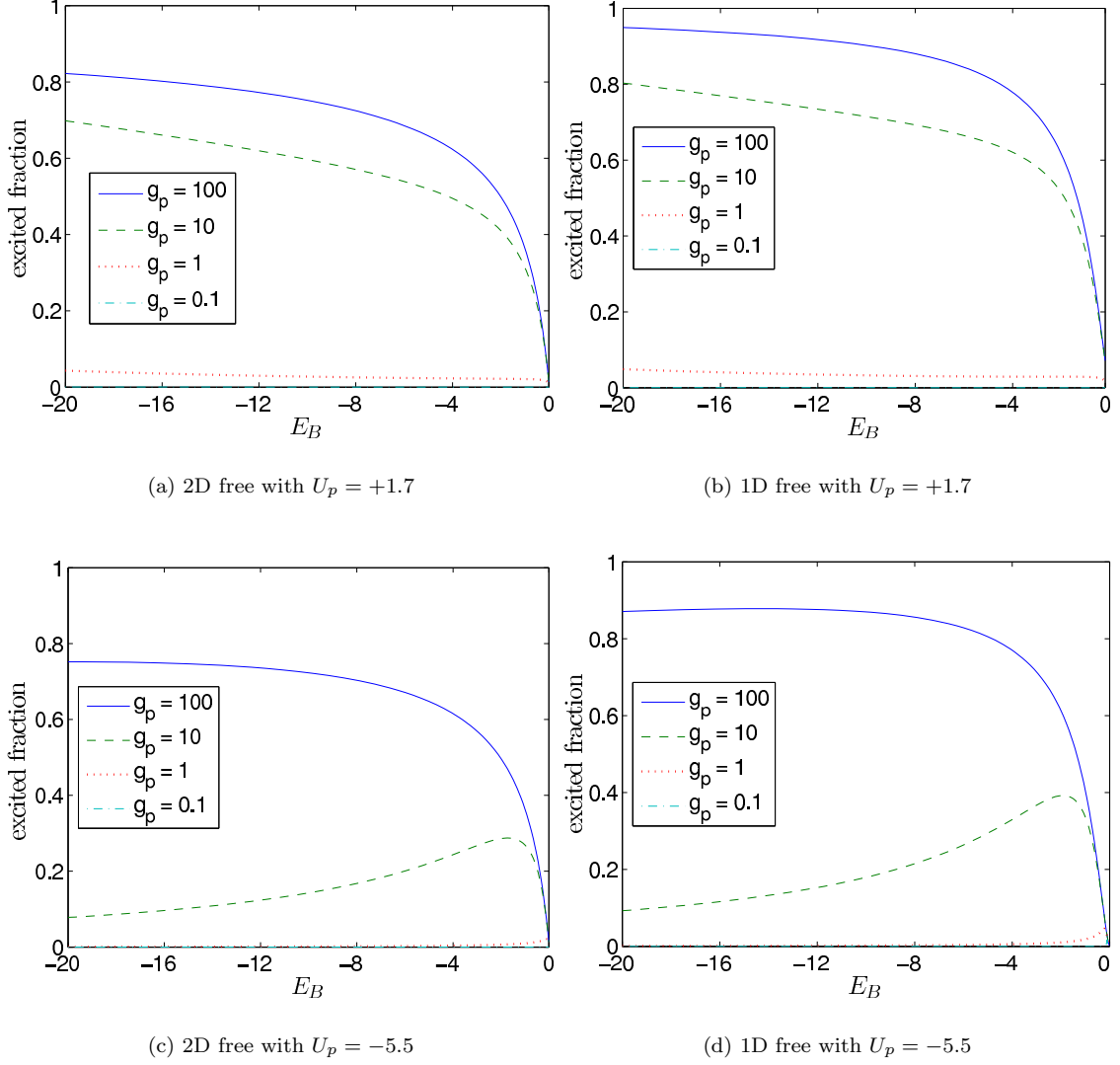


FIG. 5: Excited fraction vs. binding energy for various coupling rates.

limit [18, 19]. So the excited fraction  $P_{ex}$  continuously goes up as one increases the negative detuning. From this calculation, it is clear that for the entire region near resonance, which is of particular experimental interest, one cannot neglect the transverse excited fractions for any atoms with a wide Feshbach resonance.

The condition  $P_{ex} \ll 1$  can only be satisfied in the near resonance region for the narrow Feshbach resonance. In Fig. 5, we show the excited fractions for various atom-molecule coupling rates. The background scattering length still takes the positive or negative values corresponding to  $^{40}\text{K}$  or  $^6\text{Li}$  atoms, but we vary the coupling rate  $g_p$  (so the resonance width  $W$  changes). One can see that the condition  $P_{ex} \ll 1$  is satisfied in the whole region only when  $g_p < 1$ . As  $g_p \propto \sqrt{W}$ , the condition  $g_p < 1$  requires a very narrow resonance with the resonance width  $W < 0.01G$ . On the other hand, one can also see from the figure that to satisfy  $P_{ex} \ll 1$ , the back-

ground interaction  $|U_p|$  can be somewhat larger than 1, but it cannot be arbitrarily larger. We have tested (not shown in the figure) that the condition  $P_{ex} \ll 1$  breaks down when  $U_p$  is on the order of a few tens.

The above calculations are done with a fixed trap frequency  $\omega \simeq 2\pi \times 62 \text{ kHz}$ , as it is typical for current experiments [3]. One may expect that if the trap frequency further increases, it will become much easier to suppress the transverse excited fraction and to satisfy the condition  $P_{ex} \ll 1$ . To test whether this is true, we show in Fig. 6 the transverse excited fraction versus the magnetic field detuning for various magnitudes of the trap frequency. One can see that even if the trap frequency increases by several orders of magnitudes, the transverse excited fraction  $P_{ex}$  does not change much and remains significant. To understand this puzzling effect, we note that although the transverse level spacing increases a lot, the effective atom-molecule coupling rate also increases.

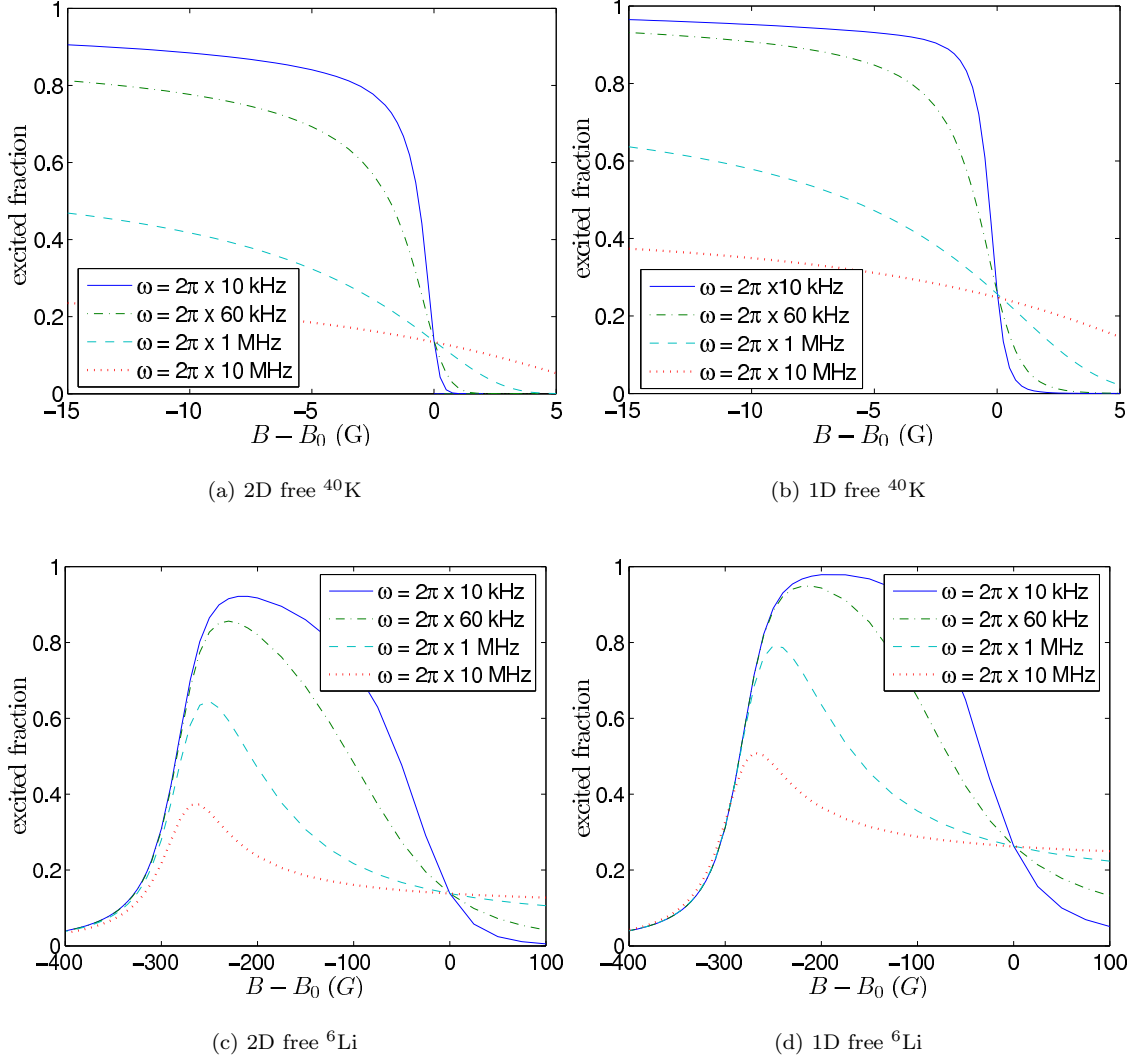


FIG. 6: Excited fraction vs. detuning for various trapping frequencies.

One cannot assume a constant atom-molecule coupling rate as an enhanced trapping in the transverse direction will suppress the pair size in all directions including the free dimensions (see the next paragraph). The effective atom-molecule coupling rate increases significantly when the pair size shrinks. As a net effect, the ratio between the atom-molecule coupling and the transverse level spacing is only a slowly-varying function of the trap frequency. The dimensionless parameter  $g_p$ , which measures the effective atom-molecule coupling strength in the unit of the trap frequency, actually characterizes this ratio. From its expression, one can verify that  $g_p$  goes as  $\omega^{-1/4}$ . So, the effective ratio  $g_p$  only drops by a factor of 10 (it will still be larger than 1 for  ${}^6\text{Li}$  and  ${}^{40}\text{K}$ ) even if the trap frequency increases by four orders of magnitudes from its current value (which is almost impossible). This explains the relative insensitivity of the excited fraction  $P_{ex}$  to the trap frequency. We can also conclude here

that for any reasonable trap strength one can imagine, it is impossible to neglect the transverse excited fractions for realistic atoms such as  ${}^6\text{Li}$  and  ${}^{40}\text{K}$  with a wide Feshbach resonance.

With an increased transverse trapping, it is easy to understand that the pair size along that direction will be suppressed. However, it is not so obvious that the pair size along the free dimension(s), where there is no trap, will correspondingly shrink. The latter actually comes from the interaction effect. Under strong interaction, the pair size cannot change only along one direction. To see the pair size shrinking along the untrapped directions, we can take the Fourier transform of the pair wavefunction  $\eta_{\mathbf{m}\mathbf{k}}$  with respect to the momentum  $\mathbf{k}$  in the free dimension(s). This Fourier transform, denoted as  $\eta_{\mathbf{m}\mathbf{n}r}$ , yields the real-space wavefunction, which is the probability amplitude for an atom pair in transverse modes  $\mathbf{m}$  and  $\mathbf{n}$  to be spaced a distance  $r$  apart in free dimension(s). From

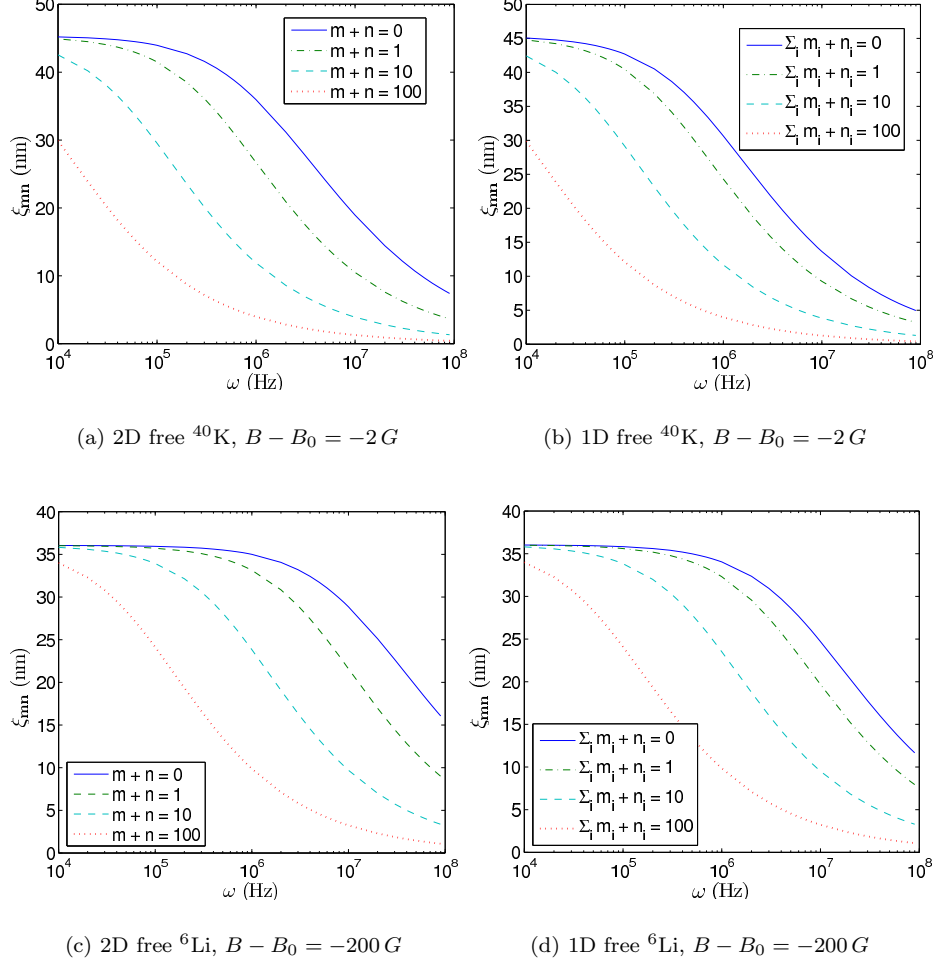


FIG. 7: Characteristic atomic pair size vs. trapping frequency for a fixed detuning.

Eq. (11),  $\eta_{\mathbf{m}\mathbf{n}r}$  is given by

$$\eta_{\mathbf{m}\mathbf{n}r} = \begin{cases} -\frac{g_p}{2\pi} (a_t L)^{-1} \beta \gamma_{\mathbf{m}\mathbf{n}} K_0(|r|/\xi_{\mathbf{m}\mathbf{n}}) & d = 2 \\ -\frac{g_p}{2} \frac{\xi_{\mathbf{m}\mathbf{n}}}{a_t^3 L} \beta \gamma_{\mathbf{m}\mathbf{n}} e^{-|r|/\xi_{\mathbf{m}\mathbf{n}}} & d = 1 \end{cases}, \quad (16)$$

where  $K_0(x) = \int_0^\infty dt \cos(xt) / \sqrt{t^2 + 1}$  is a modified Bessel function of the second kind, and

$$\xi_{\mathbf{m}\mathbf{n}} = \frac{a_t}{\sqrt{-E_B + \sum_{i=1}^{3-d} (m_i + n_i)}}, \quad (17)$$

which characterizes the pair size in the free dimension(s). The dependence of  $\xi_{\mathbf{m}\mathbf{n}}$  on the trapping frequency is shown in Fig. 7 for different transverse modes ( $\mathbf{m}\mathbf{n}$ ) on the BEC side. Note that under typical configurations, the atom population is broadly distributed over the transverse modes (as illustrated by Fig. 3), so the exact pair size in the free dimension(s) should come from the average of  $\xi_{\mathbf{m}\mathbf{n}}$  over different ( $\mathbf{m}\mathbf{n}$ ). From Fig. 7, one can see that for typical values of ( $\mathbf{m}\mathbf{n}$ ), the pair size significantly shrinks with increase of the trap frequency,

so its average will follow the same trend. As one moves towards the BCS side of resonance, the trend only becomes more dramatic. This provides a physical interpretation of the coupling enhancement: The transverse trap raises the minimum energy of the atoms relative to the molecules. Thus, the smaller atom pairs which have stronger mixture with the molecules become more energetically favorable. The net result is a sort of induced pair-wise confinement along the free dimension(s), and this increased local density causes the effective coupling to increase.

#### IV. SUMMARY

In summary, we have shown that for experimentally relevant cases, atoms trapped along one or two dimensions cannot be considered to be in the transverse ground level except when well out of the BCS-BEC crossover region. In the crossover region and for the ground state of the system, a significant fraction of atomic population



resides in the excited transverse levels, even if the gas is very dilute. Furthermore, one cannot effectively suppress the transverse excited fraction by raising the trap intensity. Even with an extremely strong trap far beyond the current experimental technology, the transverse excited fraction is not negligible yet for realistic atoms such as  $^6\text{Li}$  and  $^{40}\text{K}$  across a wide Feshbach resonance. The conclusion here is that in the experimentally interesting region, one cannot describe strongly interacting atoms under a transverse trap as a low-dimensional system by assuming a fixed transverse mode. Although this result does not

exclude an *effective* low-dimensional description of this strongly interacting system, it indeed shows that the effective description will become much more subtle, and such a description needs to take into account the broad population distribution of the atoms in all the transverse modes. The derivation of such an effective description will be a topic for further investigation [12].

This work was supported by the NSF award (0431476), the ARDA under ARO contracts, the A. P. Sloan Foundation.

- 
- [1] B. Paredes, A. Widera, V. Murg, O. Mandel, S. Fölling, I. Cirac, G. Shlyapnikov, T. Hansch, and I. Bloch, *Nature* **429**, 277 (2004).
  - [2] T. Kinoshita, T. Wenger, and D. Weiss, *Science* **305**, 5687 (2004).
  - [3] M. Kohl, T. Stoeferle, K. Guenter, M. Koehl, Tilman Esslinger, *Phys. Rev. Lett.* **94**, 080403 (2005); T. Stoeferle, H. Moritz, K. Guenter, Michael Koehl, T. Esslinger, *Phys. Rev. Lett.* **96**, 030401 (2006).
  - [4] For a theoretical review, see Y. Castin, *cond-mat/0407118*.
  - [5] D. E. Sheehy, Leo Radzihovsky, *Phys. Rev. Lett.* **95**, 130401 (2005).
  - [6] E. Orignac, R. Citro, *cond-mat/0601269*.
  - [7] M. Olshanii, *Phys. Rev. Lett.* **81**, 938 (1998); T. Busch, B.-G. Englert, K. Rzazewski, M. Wilkens, *Found. Physics* **28**, 549 (1998); D.S. Petrov, M. Holzmann, G.V. Shlyapnikov, *Phys. Rev. Lett.* **84**, 2551 (2000); P. O. Fedichev, M. J. Bijlsma, P. Zoller, *Phys. Rev. Lett.* **92**, 080401 (2004).
  - [8] M. Greiner, *et al.*, *Nature* **415**, 39 (2002); C. Orzel, *et al.*, *Science* **291**, 2386 (2001); D. Jaksch, *et al.*, *Phys. Rev. Lett.* **81**, 3108 (1998).
  - [9] C.A. Regal, M. Greiner and D.S. Jin, *Phys. Rev. Lett.* **92**, 040403 (2004); M.W. Zwierlein *et al.*, *Phys. Rev. Lett.* **92**, 120403 (2004); C. Chin *et al.*, *Science* **305**, 1128 (2004).
  - [10] L.-M. Duan, *Phys. Rev. Lett.* **95**, 243202 (2005).
  - [11] R. B. Diener, T.-L. Ho, *Phys. Rev. Lett.* **96**, 010402 (2006).
  - [12] J. P. Kestner, L.-M. Duan, in preparation.
  - [13] D. B. M. Dickerscheid, H. T. C. Stoof, *cond-mat/0506530*; D. B. M. Dickerscheid *et al.*, *Phys. Rev. A* **71**, 043604 (2005).
  - [14] M. Holland *et al.*, *Phys. Rev. Lett.* **87**, 120406 (2001); E. Timmermans, *et al.*, *Phys. Rep.* **315**, 199 (1999).
  - [15] Q. Chen, J. Stajic, S. Tan, K. Levin, *Physics Reports* **412**, 1 (2005).
  - [16] We will let this energy cutoff  $E_c$  tend to infinity in our final equations as the low-energy physics there is independent of the value of the high energy cutoff (it cancels with the divergence of  $S(E)$  in Eq. (13)).
  - [17] M. Bartenstein *et al.*, *Phys. Rev. Lett.* **94**, 103201 (2005).
  - [18] W. Yi, L.-M. Duan, *cond-mat/0603264*.
  - [19] M. H. Szymanska, K. Goral, T. Koehler, K. Burnett, *Phys. Rev. A* **72**, 013610 (2005).